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VALIDATION OF SINUSOIDAL MODEL FOR SIMULATION OF DIURNAL TEMPERATURES

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ABSTRACT : For many agricultural and climatological purposes it is useful to obtain an approximation of the diurnal screen temperature curve for a particular place and time of year. The objective of this work is to determine the accuracy of sinusoidal model (De Wit et al., 1978) for reconstructing hourly temperature variation. Inputs for the model are daily maximum and minimum temperatures and time of sunrise. The model uses two different cosine functions for the period between times of minimum and maximum temperatures and between times of maximum and minimum temperatures of the next day. The model was fitted to the data of 144 days randomly selected from each month and year (1991-93) at Pune University observatory. The model was evaluated for accuracy by comparing observed and simulated diurnal temperatures in terms of the sum of residuals, sum of absolute value of residuals and root mean square of error statistics. The results present a reasonable fit based on the overall diurnal trends. Limited success of the model on overcast days notwithstanding, it can be profitably used in growth models particularly of rainfed crops to predict their yields.

INTRODUCTION

Temperature is an important weather factor affecting biological processes. The rate of growth, development and survival of most organisms is temperature dependent. Biological response to temperature is non-linear (Wann et al. 1985) and development has been demonstrated to occur at a rate proportional to biological time i.e. proportional to accumulation of 'hour degrees' or 'day degrees' above a certain temperature threshold (Bernhardt and Shephard, 1978). Systematic temperature variations on diurnal basis, therefore, are required in the input to agricultural models, particularly crop growth models.

In most of the surface meteorological observatories daily minimum and maximum air temperatures are commonly reported. The ambient air temperature varies in a somewhat regular pattern reflecting both the annual and diurnal cycles of extra-terrestrial radiation. Since these cycles can be described adequately it should be possible to have an empirical model which is capable of restructuring mean diurnal temperature variation from maximum and minimum temperatures for a particular location and time of the year.

Temperature curves are a combination of periodic sine and exponential decay curves. They are not readily represented by a few terms of a Fourier series, and furthermore such methods ignore the basic physical relationships of the diurnal cycle. The two most frequently used techniques for simulating hourly temperatures are energy budget models and empirical models. Energy budget models have proved more accurate than simple curve fitting models but have disadvantage of requiring data on many climatological variables and heat transfer parameters. Fluker (1958) and Parton (1978) developed empirical models by using Fourier heat conduction equation which requires only maximum and minimum temperatures (Parton and Logan, 1981). Several researchers tried to predict diurnal variation in temperature by using curve fitting models like linear (Saunders, 1975), non-linear (Heuer et al. 1978), Fourier series (Carson, 1963; Walter, 1967 and Hansen and Driscoll, 1977). sine exponential (Parton and Logan, 1981) and sinusoidal (De Wit et al. 1978). Wann et al (1985) compared several mathematical models mentioned above and concluded that Sinusoidal model was the best one. This study makes an attempt to simulate hourly temperatures by using sinusoidal model.

METHODOLOGY : SINUSOIDAL MODEL

The model was initially presented by De Wit et al. (1978) and was obtained from the subroutine WAVE in ROOTSIMU V4.0 given by Hoogenboom and Huck (1986). The model uses minimum temperature of the next day and divides the day into two segments. The one is from sunrise to 1400 hour and the other from 1400 hour to sunrise of the following day. The model assumes that -

i. maximum temperature occurs at 1400 hour,
ii. minimum temperature occurs sometime around sunrise
iii. and a cosine function describes the variation for the period from the time of minimum temperature to the time of maximum temperature and another cosine function for the period from the time of maximum temperature to the time of minimum temperature of the next day.

The intervening temperatures are calculated from the following equations :

For [illegible] $< H < RISE$ and $1400h < H < 2400h$

$$T(H) = TAVE + AMP\,(\cos\{H'/(10.0 + RISE)]$$

For $RISE < H < 1400h$

$$T(H) = TAVE - AMP\,(\cos\{(H-RISE)/(14-RISE)\}]$$

where

$RISE$ = the time of sunrise in hours

$T(H)$ = the temperature at any hour, H = in hours

$H' = H + 0$ if $H < RISE$, $H' = H - 14$ if $H > 1400h$

$TAVE = (Tmin + Tmax)/2$, $= (Tmax - Tmin)/2$

DATA SOURCES

The air temperature data used to develop sinusoidal model for calculating hourly temperatures were collected from the University of Pune observatory (18°32' N, 73° 51E and 559 metres height). Randomly selected 3 days from each month and year (total 144 days) for the period of 1991 - 93 (3 years) formed the data base. For validation, the hourly observed temperature values for the same days were collected from the thermograph records. The information on time of sunrise for different months in the year is obtained from the India Meteorolgical Department.

ERROR ANALYSIS

To test the model, measured and simulated temperatures at hourly interval were compared. The 'goodness of fit' was assessed in several ways. The overall accuracy of the shape of the simulated curve was examined by calculated root mean square error (RMSE). When the predicted temperatures were closer to the observed ones, the RMSE was smaller. It is a better error indicator since it tends to penalize large individual errors heavily.

$$RMSE = \left[\sum_{i=1}^{n} (Toi - Tei)^2/n \right]^{1/2}$$

The sum of residuals (RES) and the sum of the absolute value of residuals (IRESI) can be used to determine the tendency of the model to consistently overpredict or under predict the temperature over a given period. After comparison of RES and IRESI it is possible to determine how errors in the model cancel over time. A large positive RES that approaches IRESI suggests that the model consistently underestimates the actual temperature whereas a large negative RES compared to IRESI denotes a tendency for the model to overestimate the observed value. A small RES vis-a-vis IRESI indicates that the errors in the model cancel over the 24-hour period.

RESULTS AND DISCUSSION

The diurnal change in mean air temperature near the earth's surface is a result of complicated energy exchange processes which involve solar heating during the day and radiative cooling during the night (Oke, 1978). The air temperature exhibits a regular pattern of variation that follows diurnal cycles of solar radiation. Superimposed on these regular cycles are fluctuations of variable duration and amplitude created by changing weather conditions like cloudiness, rain, warm and cold spells etc. These fluctuations have been removed from the data and the regular cycles have been isolated and studied by using monthly diurnal data averaged for the years 1991, 92 and 93. Figure 1 shows that the model performs well in simulating the mean monthly diurnal temperature and that seasonal changes in the pattern are well represented in the model. A summary of statistical comparisons of observed and simulated temperatures are presented in Table 1.

Table 1

Statistical comparisons of diurnal temperature simulations

Months	R^2	RMSE	RES	IRESI
January	0.90	2.41	–6.47	45.13
February	0.95	3.19	29.83	60.93
March	0.98	2.13	29.23	38.13
April	0.98	2.22	32.60	42.26
May	0.95	2.02	8.73	37.73
June	0.97	2.20	8.53	35.53
July	0.74	1.46	–7.27	28.86
August	0.92	1.25	–10.50	24.90
September	0.91	1.43	8.77	29.06
October	0.67	1.96	–7.07	37.83
November	0.91	2.15	–2.20	41.50
December	0.99	2.48	5.20	50.33

The table given above indicates that the RMSE varied from a low of 1.25 for August to 3.19 for February. A large positive RES values approaching IRESI for February, March and April suggest that the model consistently underestimates the observed temperature. A small RES compared to IRESI for the rest of the months indicates that the errors in the model tend to cancel over the 24-hour period.

The high $R^2 > 0.9$ for most of the months presents a reasonable fit, based on the overall diurnal trends. However a very low correlation coefficient is reported for July and October marking variable diurnal trends mostly resulting from prevalence of rains and cloudy conditions. Effect of solar radiation on temperature is obvious. The accuracy of model improves in inverse proportion to cloud cover. The model is constructed to produce the classic diurnal

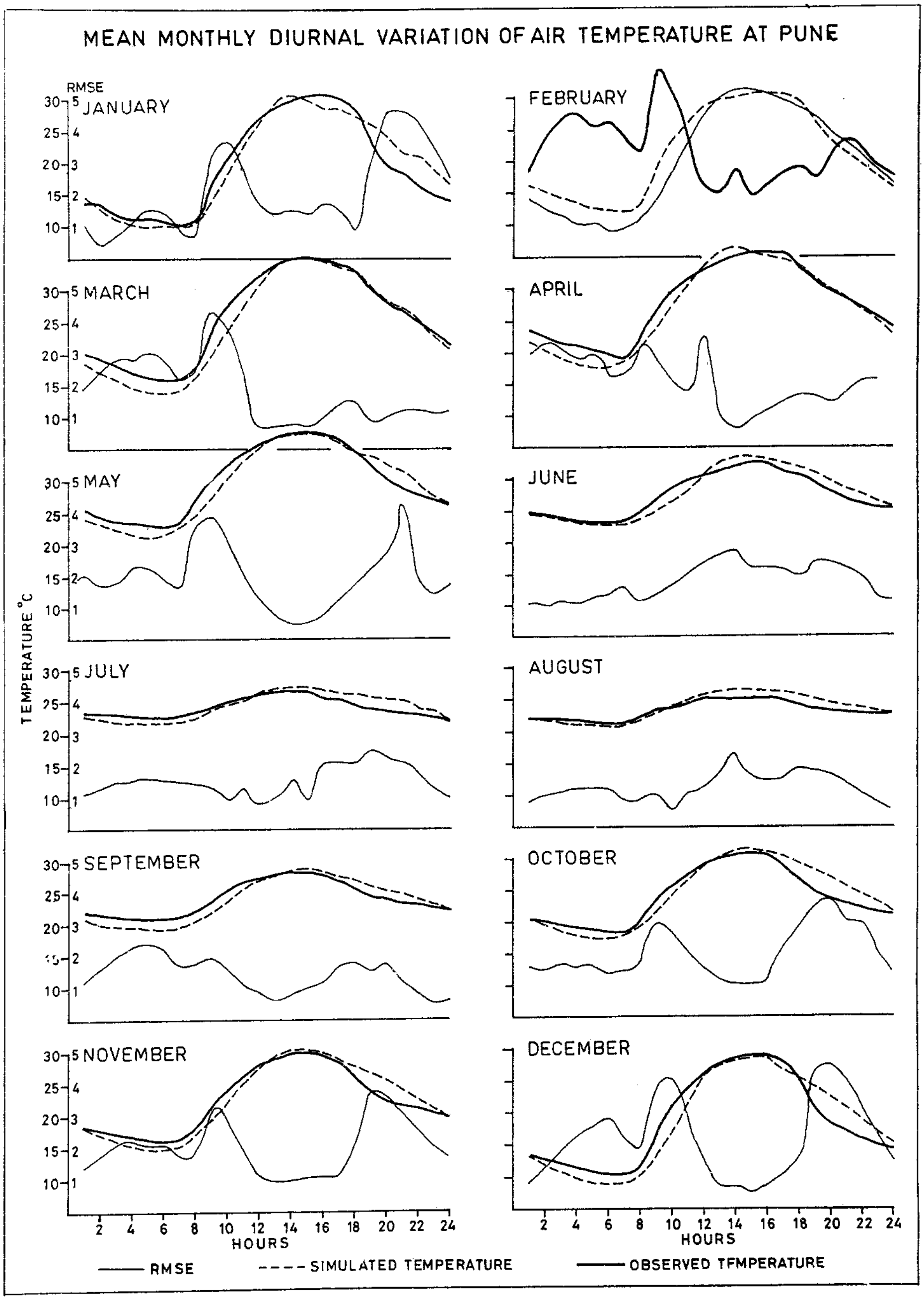

Fig. 1 : Mean monthly diurnal variation of air temperature at Pune

rhythm of temperature. Such a rhythm is most clearly expressed on sunny days when terrestrial irradiation follows a sinusoidal path. On cloudy days factors other than radiation assume more importance in determining air temperature and alter the shape of the diurnal temperature curve. The low RMSE i.e. below 2.0 for July, August, September and October goes with a good representation of a diurnal temperature trend (Figure 1). Low RMSE and low correlation coefficient values for monsoon months may be function of platykurtic form and irregular nature of the diurnal temperature trends on monsoon days respectively. It is important to note that for most of the months during afternoon hours, the model has a tendency to overpredict the actual value and during the period from 9 to 10 and 19 to 21 hours the larger RMSE, RES and IRESI values are reported. The detailed analysis of the hourly distribution of error parameters revealed that this may be due to the fact that the sinusoidal model does not incorporate the information on sunset time as input. This reduces the flexibility of model for data fitting by not providing an appropriate description of the cooling process after sunset. The model implicitly assumes that maximum cooling rate always occurs at the mid-point between the times of maximum and minimum temperatures. In the same manner, it assumes that maximum heating rate is observed at mid-point between the times of minimum and maximum temperatures. The low values of RMSE are noted during the mid-day hours (1200 to 1600 hours) around the time of maximum temperature. Thus the rates of heating and cooling and the resultant form of temperature curve are completely determined by the times and magnitudes of these temperature extremes.

The cursory examination of the hourly observed and simulated temperatures reveals that from midnight to about noon time the temperatures are under estimated whereas during the remaining period of the day they are overestimated. The importance of simulated hourly temperatures is related to physiological processes involved in plant growth models. The large day-time errors in diurnal temperature may wrongly simulate photosynthesis and respiration which are at maximum during the day. For modelling crop respiration and frost resistance accurate simulation of night-time temperature is necessary. The slight over assessment of photosynthesis and transpiration from sunrise to mid-day made by the model is counterbalanced by slight underestimation of the same from mid-day to sunset time. However, the large errors in simulation after sunset during winter months from October through January indicate the inability of the model to predict frostproneness of the area.

An example of July 16, 1991 is illustrated (Figure 2-B) when a model worked reasonably well for calculating hourly temperatures. This day indicated a smooth diurnal trend throughout the 24-hour period and the range between maximum and minimum temperatures was small (4.5°C). The maximum temperature occurred at 1400 hour while minimum at the time of sunrise; thus most of the assumptions of the model were met. The simulated results for all the hours are reasonably close to the observed values. The difference between estimated and observed hourly temperatures (Figure 2-C) clearly indicates that the model under predicts till the time of Maximum temperature and over protects afterwards. The error parameters for the day are summarised in Table 2. In this case, the R^2 is as high as 0.92, the RMSE, RES and IRESI

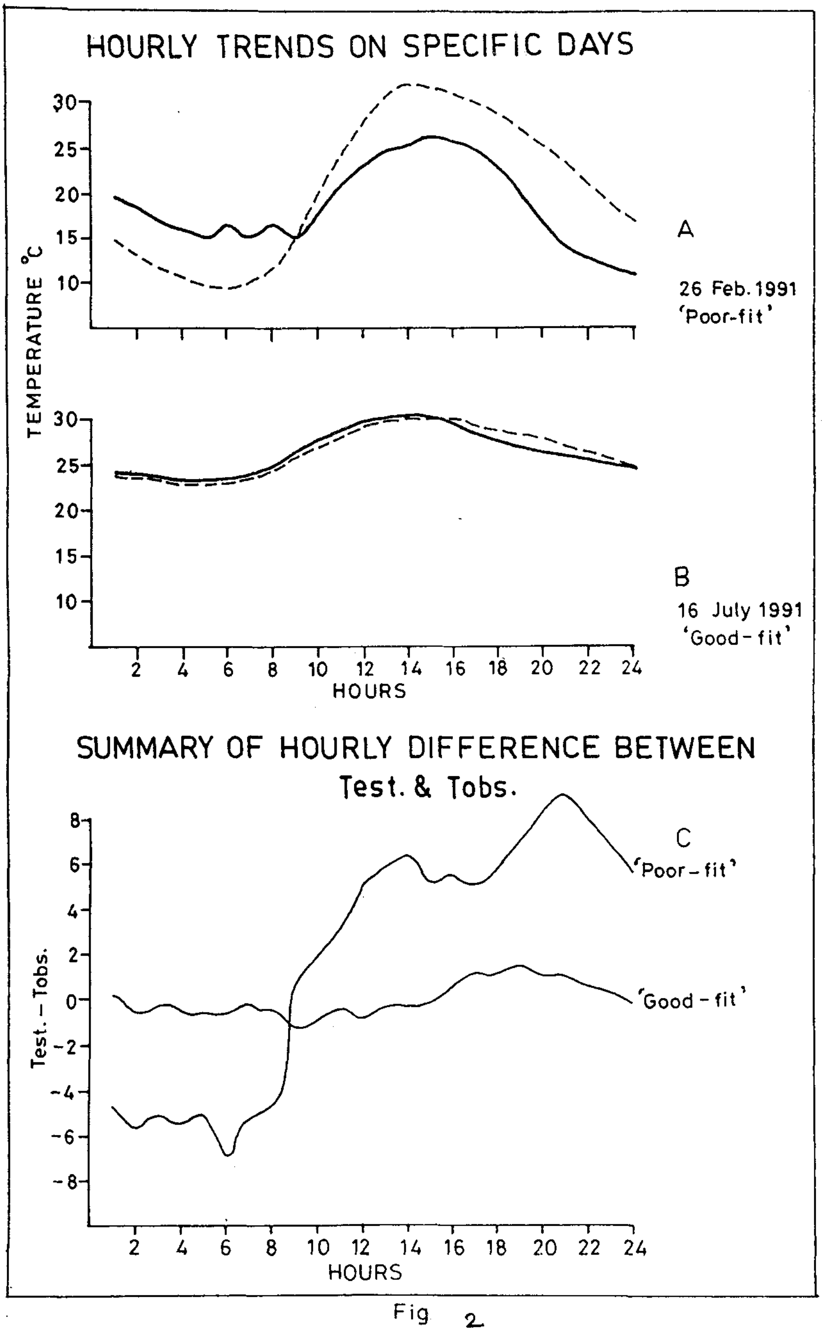

Fig. 2 : Summary of Hourly difference between test & Tobs

are small suggesting a 'good fit' for the observed data.

Table 2

Statistical summary of magnitude of errors on selected days

Day	R^2	RMSE	RES	IRESI
July 16,1991 Day of 'good fit'	0.92	0.72	-6.0	14.40
Feb. 26, 1991 Day of 'poor fit'	0.55	5.78	-47.10	131.50

A second example of February 26, 1991 is indicated in figure 2-A. This day represents a relatively 'poor fit' of the sinusoidal model for computing hourly temperatures. The trend of observed data was not smooth and temperature difference between maximum and minimum was large (11.4°C). The lowest temperature occurred one hour before sunrise and the highest reached at 1500 hour. Since most of the assumptions of the model failed, the trends throughout the 24-hour period were not well fitted to the observed temperatures. The largest positive difference between estimated and observed temperatures (Figure 2-C) is noted 9°C for 2100 hour and the negative one is 6.5°C for 0600 hour. The statistical summary of error parameters (Table 2) clearly points out that correlation coefficient is low (0.55) as compared to that of the previous example. The large negative RES describes the tendency of over prediction of the model.

CONCLUSIONS

The model provides reasonable estimates of the diurnal variation of temperatures using cosine functions. The results are predictably better for long-term averages since extremes tend to be smoothed as more data are averaged. The better agreement between observed and estimated temperatures on clear days is disconcerting on cloudy days. The portability of the model strongly depends on the time of maximum temperature assumed and thus it is location specific.

The method fails to assess the frostproneness of the region during a rabi season as indicated by over prediction on night temperatures. However, during monsoon months slight under prediction after sunrise balances afternoon over prediction of the observed values. In addition, the magnitude of error is minimum during that period which is a main crop growing season in India. The model, therefore, can be profitably used to monitor growth of rainfed crops and to predict crop yields.

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